P2-CONNECTEDNESS IN L-TOPOLOGICAL SPACES

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Abstract. In this paper, a certain new connectedness of L-fuzzy subsets in L-topological spaces is introduced and studied by means of preclosed sets. It preserves some fundamental properties of connected set in general topology. Especially the famous K. Fan's Theorem holds.

1. Introduction

Connectivity is one of the important notions in general topology. There have been many works about connectedness in L-topological spaces (see [1, 2, 3, 4, 7, 10, 12, 13, 14, 15, 16, 17] etc.). In [4], Bai introduced P-connectedness in L-topological spaces by means of preclosed sets.

In this paper, based on [15], we shall introduce a certain new connectedness in L-topological spaces by means of preclosed sets, which is called P2-connectedness. The P2-connectedness preserves some fundamental properties of connected sets in general topology. Especially the famous K. Fan's Theorem holds for P2-connectedness. We also shall discuss the relation between P-connectedness in [4] and P2-connectedness.

Throughout his paper $(L, \bigvee, \bigwedge, ')$ is a completely distributive de Morgan algebra, X a nonempty set. L^X is the set of all L-fuzzy sets on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$.

An element a in L is called prime element if $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. An element a in L is called a co-prime element if a' is a prime element [6]. The set of all nonzero co-prime elements in L is denoted by $M(L)$. The set of all nonzero co-prime elements in L^X is denoted by $M(L^X)$. For an L-fuzzy set D, $M(D)$ denotes the set of all nonzero co-prime elements contained in D.

An L-topological space (or L-space for short) is a pair $(X, \mathcal{T})$, where $\mathcal{T}$ is a subfamily of L^X which contains $\underline{0}$, $\underline{1}$ and is closed for any suprema and finite infima. $\mathcal{T}$ is called an L-topology on X. Each member of $\mathcal{T}$ is called an open L-set and its quasi-complementation is called a closed L-set.

In an L-space (X, δ), an L-fuzzy set A is called a preclosed set if $A \geq A^{\circ -}$. The pre-closure of the L-fuzzy set A is the intersection of all preclosed sets containing A. It is denoted by $pcl(A)$. Two L-fuzzy sets A, B are called P-separated if $pcl(A) \wedge B = A \wedge pcl(B) = \underline{0}$ [4].

Definition 1.1. [4] In an L-space (X, δ), an L-fuzzy set D is called P-connected if D cannot be represented as a union of two P-separated non-null sets.

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Definition 1.2. [4] For two L-spaces (X,δ) and (Y,τ), a mapping $f : X \to Y$ is called a P-irresolute mapping if $f^{\leftarrow}(B)$ is preclosed in (X,δ) for each preclosed set B in (Y,τ).

Lemma 1.3. [15] *Let $A, B \in L^X$ and $A \not\leq B$. If $1 \in M(L)$, then $A' \vee B \neq \underline{1}$.*

Definition 1.4. [15] In an L-space (X,δ), an L-fuzzy set D is called connected if there don't exist preclosed sets A, B such that

$$D \not\leq A, D \not\leq B, D' \vee A \vee B = \underline{1}, D \wedge A \wedge B = \underline{0}.$$

2. P2-connectedness of L-fuzzy sets

Definition 2.1. Let (X,δ) be an L-space, $D \in L^X$. D is called P2-connected if there don't exist preclosed sets A, B such that

$$D \not\leq A, D \not\leq B, D' \vee A \vee B = \underline{1}, D \wedge A \wedge B = \underline{0}.$$

The following example is non-trivial for P2-connectedness.

Example 2.2. Let $X_1 \cap X_2 = \emptyset$, $X = X_1 \cup X_2$, $L = [0,1]$. Defined fuzzy set $(C_a, C_b) \in [0,1]^X$ as follows:

$$(C_a, C_b)(x) = \begin{cases} a, & x \in X_1; \\ b, & x \in X_2. \end{cases}$$

Take

$$\gamma = \{(C_{0.4}, C_1), (C_1, C_{0.4}), (C_{0.5}, C_0), (C_0, C_{0.5})(C_{0.7}, C_0), (C_0, C_{0.7})\}.$$

Let δ be a [0,1]-topology generated by γ on X. Now we prove that $(C_{0.5}, C_{0.5})$ is P2-connected. In fact, suppose that $(C_{0.5}, C_{0.5})$ is not P2-connected. Then there exist two preclosed sets A, B such that

$$(C_{0.5}, C_{0.5}) \not\leq A,\ (C_{0.5}, C_{0.5}) \not\leq B, (C_{0.5}, C_{0.5})' \vee A \vee B = \underline{1},\ (C_{0.5}, C_{0.5}) \wedge A \wedge B = \underline{0}.$$

This implies that

$$(C_{0.5}, C_{0.5}) \not\leq A,\ (C_{0.5}, C_{0.5}) \not\leq B, A \vee B = \underline{1},\ A \wedge B = \underline{0}.$$

Obviously A (or B) satisfying $A \wedge B = \underline{0}$ must be in $\{(C_a, C_0), (C_0, C_a) \mid a \in [0.6, 0.7)\}$. But any two preclosed sets in $\{(C_a, C_0), (C_0, C_a) \mid a \in [0.6, 0.7)\}$ don't satisfy $A \vee B = \underline{1}$. Therefore $(C_{0.5}, C_{0.5})$ is P2-connected.

The following theorem gives some characterizations of P2-connectedness.

Theorem 2.3. *Let (X,δ) be an L-space, $D \in L^X$. If $1 \in M(L)$, then the following conditions are equivalent.*

(1) *D is P2-connected.*

(2) *There don't exist preclosed sets A, B such that*

$$D \wedge A \neq \underline{0},\ D \wedge B \neq \underline{0},\ D' \vee A \vee B = \underline{1},\ D \wedge A \wedge B = \underline{0}.$$

(3) *There don't exist preopen sets U, V such that*

$$D \not\leq U,\ D \not\leq V,\ D' \vee U \vee V = \underline{1},\ D \wedge U \wedge V = \underline{0}.$$

(4) *There don't exist preopen sets U, V such that*

$$D \wedge U \neq \underline{0},\ D \wedge V \neq \underline{0},\ D' \vee U \vee V = \underline{1},\ D \wedge U \wedge V = \underline{0}.$$

Proof. (1) $\Rightarrow$ (2). Suppose that there exist preclosed sets A, B such that

$$D \wedge A \neq \underline{0},\ D \wedge B \neq \underline{0},\ D' \vee A \vee B = \underline{1},\ D \wedge A \wedge B = \underline{0}.$$

Then obviously we have that

$$D' \vee A \vee B = \underline{1},\ D \wedge A \wedge B = \underline{0}.$$

In this case, we can prove that $D \not\leq A,\ D \not\leq B$. In fact, if $D \leq A$, then $D \wedge A \wedge B = D \wedge B = \underline{0}$, which contradicts $D \wedge B \neq \underline{0}$.

(2) $\Rightarrow$ (3). Suppose that there exist preopen sets U, V such that

$$D \not\leq U,\ D \not\leq V,\ D' \vee U \vee V = \underline{1},\ D \wedge U \wedge V = \underline{0}.$$

Put $A = U'$, $B = V'$. Obviously we have that

$$D' \vee A \vee B = \underline{1},\ D \wedge A \wedge B = \underline{0}.$$

In this case, we can prove that $D \wedge A \neq \underline{0},\ D \wedge B \neq \underline{0}$. In fact, if $D \wedge A = \underline{0}$, then $D' \vee U = D' \vee A' = \underline{1}$, hence by Lemma 1.3 we obtain that $D \leq U$, which contradicts $D \not\leq U$.

(3) $\Rightarrow$ (4) is analogous to (1) $\Rightarrow$ (2).

(4) $\Rightarrow$ (1) is analogous to (2) $\Rightarrow$ (3). □

Theorem 2.4. *Let D be a P2-connected set in (X, δ). If $D \leq E \leq pcl(D)$, then E is also a P2-connected set.*

Proof. Suppose that E is not P2-connected. Then there exist two preclosed sets A, B such that

$$E \not\leq A, E \not\leq B, E' \vee A \vee B = \underline{1}, E \wedge A \wedge B = \underline{0}.$$

Hence

$$D' \vee A \vee B = \underline{0},\ D \wedge A \wedge B = \underline{0}.$$

In fact, we also have that $D \not\leq A, D \not\leq B$ (If $D \leq A$, then $E \leq pcl(D) \leq A$, which contradicts $E \not\leq A$). This shows that D is not P2-connected in (X, δ), which contradicts that D is P2-connected. Therefore E is P2-connected. □

Lemma 2.5. *Let (X, δ) be an L-space and $D, E \in L^X$. Then D, E are P-separated if and only if there exist two preclosed sets A, B such that $D \leq A,\ E \leq B$ and $(D \vee E) \wedge A \wedge B = \underline{0}$.*

Proof. ($\Leftarrow$) If there exist two preclosed sets A, B such that $D \leq A,\ E \leq B$ and $(D \vee E) \wedge A \wedge B = \underline{0}$, then we have that

$$(D \wedge pcl(E)) \vee (pcl(D) \wedge E) \leq (D \wedge B) \vee (E \wedge A) = (D \vee E) \wedge A \wedge B = \underline{0}.$$

This shows that D, E are P-separated.

($\Rightarrow$) If D, E are P-separated, then $(D \wedge pcl(E)) \vee (pcl(D) \wedge E) = \underline{0}$. Take $A = pcl(D)$ and $B = pcl(E)$. Then

$$(D \vee E) \wedge A \wedge B = (D \vee E) \wedge pcl(D) \wedge pcl(E) = (D \wedge pcl(E)) \vee (pcl(D) \wedge E) = \underline{0}.$$

The proof is completed. □

Theorem 2.6. *Let D, E be two P2-connected L-fuzzy sets in an L-space (X, δ). If D, E are not P-separated, then $D \vee E$ is P2-connected.*

Proof. Suppose that $D \vee E$ is not P2-connected. Then there exist two preclosed sets A, B such that

$$D \vee E \not\leq A, D \vee E \not\leq B, (D \vee E)' \vee A \vee B = \underline{1}, (D \vee E) \wedge A \wedge B = \underline{0}.$$

Hence we have that

$$D' \vee A \vee B = \underline{1}, D \wedge A \wedge B = \underline{0},\ E' \vee A \vee B = \underline{1}, E \wedge A \wedge B = \underline{0}.$$

By $D \vee E \not\leq A$ we have that $D \not\leq A$ or $E \not\leq A$. Suppose that $D \not\leq A$. Then we must have that $D \leq B$ since D is P2-connected. Further by $D \vee E \not\leq B$ we obtain that $E \not\leq B$. In this case it follows that $E \leq A$. Therefore by $(D \vee E) \wedge A \wedge B = \underline{0}$ and Lemma 2.6 we know that D, E are P-separated, which contradicts that D, E are not P-separated. The proof is completed. □

Theorem 2.7. *Let $\{D_t \mid t \in \Omega\}$ be a family of P2-connected L-fuzzy sets. If there is an $s \in \Omega$ such that for each $t \in \Omega - \{s\}$, D_t and D_s are not P-separated, then $\bigvee_{t\in\Omega} D_t$ is P2-connected.*

Proof. Suppose that $\bigvee_{t\in\Omega} D_t$ is not P2-connected. Then there exist two preclosed sets A, B such that

$$\bigvee_{t\in\Omega} D_t \not\leq A, \bigvee_{t\in\Omega} D_t \not\leq B, (\bigvee_{t\in\Omega} D_t)' \vee A \vee B = \underline{1}, (\bigvee_{t\in\Omega} D_t) \wedge A \wedge B = \underline{0}.$$

Hence there exist $t_1, t_2 \in \Omega$ such that

$$D_{t_1} \vee D_{t_2} \vee D_s \not\leq A, D_{t_1} \vee D_{t_2} \vee D_s \not\leq B,$$

$$(D_{t_1} \vee D_{t_2} \vee D_s)' \vee A \vee B = \underline{1}, (D_{t_1} \vee D_{t_2} \vee D_s) \wedge A \wedge B = \underline{0}.$$

This shows that $D_{t_1} \vee D_{t_2} \vee D_s$ is not P2-connected. But by Theorem 2.7 we know that $D_{t_1} \vee D_{t_2} \vee D_s$ is P2-connected. Thus we obtain a contradiction. The proof is completed. □

Corollary 2.8. *Let $\{D_t \mid t \in \Omega\}$ be a family of P2-connected L-fuzzy sets. If $\bigwedge_{t\in\Omega} D_t \neq \underline{0}$, then $\bigvee_{t\in\Omega} D_t$ is P2-connected.*

Theorem 2.9. *Let $f : (X, \delta) \to (Y, \tau)$ be a P-irresolute mapping. If D is P2-connected in (X, δ), then so is $f(D)$ in (Y, τ).*

Proof. Suppose that $f(D)$ is not P2-connected in (Y, τ). Then there exist two preclosed sets A, B in (Y, τ) such that

$$f(D) \not\leq A,\ f(D) \not\leq B,\ f(D)' \vee A \vee B = \underline{1},\ f(D) \wedge A \wedge B = \underline{0}.$$

Thus by $D \leq f^{\leftarrow}(f(D))$ or $D' \geq f^{\leftarrow}(f(D)')$ we have that

$$D \not\leq f^{\leftarrow}(A), D \not\leq f^{\leftarrow}(B), D' \vee f^{\leftarrow}(A) \vee f^{\leftarrow}(B) = \underline{1}, D \wedge f^{\leftarrow}(A) \wedge f^{\leftarrow}(B) = \underline{0}.$$

Since $f : (X, \delta) \to (Y, \tau)$ is a P-irresolute mapping, we know that $f^{\leftarrow}(A)$ and $f^{\leftarrow}(B)$ are preclosed sets in (X, τ). This shows that D is not P2-connected in

(X, τ), which contradicts that D is P2-connected. Therefore $f(D)$ is P2-connected in (Y, τ). □

Theorem 2.10. *Let (X, δ) be an L-space and $D \in L^X$. Then D is P2-connected if and only if for any two co-prime elements $a, b \leq D$, there exists a P2-connected set E such that $a, b \leq E \leq D$.*

Proof. The necessity is obvious. Now we prove the sufficiency. Suppose that D is not P2-connected in (X, δ). Then there exist two preclosed sets A, B in (X, δ) such that

$$D \not\leq A, D \not\leq B, D' \vee A \vee B = \underline{1}, D \wedge A \wedge B = \underline{0}.$$

Take two co-prime elements $a, b \leq D$ such that $a \not\leq A$ and $b \not\leq B$. Let E be a P2-connected set satisfying $a, b \leq E \leq D$. We have that

$$E \not\leq A, E \not\leq B, E' \vee A \vee B = \underline{1}, E \wedge A \wedge B = \underline{0}.$$

This shows that E is not P2-connected in (X, δ), a contradiction. The proof is completed. □

Now we shall give K. Fan's theorem of P2-connectedness.

Definition 2.11. Let (X, δ) be an L-space, $D \in L^X$ and $\mathcal{F}$ denote the set of all preclosed sets in (X, δ). A mapping $P : M^*(D) \to \mathcal{F}$ is called a pre-remote neighborhood mapping on D, if for each $e \in M^*(D)$, it holds that $e \not\leq P(e)$.

Example 2.12. Let $X_1 \cap X_2 = \emptyset$, $X = X_1 \cup X_2$, $L = [0, 1]$. Defined fuzzy set $(C_a, C_b) \in [0, 1]^X$ as follows:

$$(C_a, C_b)(x) = \begin{cases} a, & x \in X_1; \\ b, & x \in X_2. \end{cases}$$

Let

$$\delta = \{\emptyset, \underline{1}, (C_{0.5}, C_{0.5})\}.$$

Then δ is a [0,1]-topology on X. $\forall e \in M(L^X)$, define

$$P(e) = \begin{cases} (C_{0.5}, C_{0.5}), & \text{if } e \not\leq (C_{0.5}, C_{0.5}); \\ \underline{0}, & \text{if } e \not\leq (C_{0.5}, C_{0.5}) \end{cases}$$

Then P is a pre-remote neighborhood mapping.

Theorem 2.13. *Let (X, δ) be an L-space and $D \in L^X$. Then $D$is P2-connected if and only if for any two co-prime elements $a, b \in M^*(D)$ and any pre-remote neighborhood mapping $P : M^*(D) \to \mathcal{F}$, there exist finite many co-prime elements $e_1 = a, e_2, \cdots, e_n = b$ in D such that*

$$D' \vee P(e_i) \vee P(e_{i+1}) \neq \underline{1}, \quad i = 1, 2, \cdots, n-1.$$

Proof. ($\Leftarrow$) Suppose that D is not P2-connected. Then there exist two preclosed sets A, B such that

$$D \not\leq A, D \not\leq B, D' \vee A \vee B = \underline{1}, D \wedge A \wedge B = \underline{0}.$$

Define a pre-remote neighborhood mapping $P : M^*(D) \to \mathcal{F}$ such that

$$\forall e \in M^*(D),\ P(e) = \begin{cases} A, & \text{if } e \leq B; \\ B, & \text{if } e \not\leq B. \end{cases}$$

Take $a, b \in M^*(D)$ such that $a \not\leq A$ and $b \not\leq B$. Then for arbitrary finite many co-prime elements $e_1 = a, e_2, \cdots, e_n = b$ in D, there exists an i such that $D' \vee P(e_i) \vee P(e_{i+1}) = \underline{1}$, this contradicts the condition of Theorem. Thus the sufficiency is proved.

($\Rightarrow$) Suppose that there exist two co-prime elements $a, b \in M^*(D)$ and a pre-remote neighborhood mapping $P : M^*(D) \to \mathcal{F}$ such that for arbitrary finite many co-prime elements $e_1 = a, e_2, \cdots, e_n = b$ in D, the following fact is not true:

$$D' \vee P(e_i) \vee P(e_{i+1}) \neq \underline{1}, \quad i = 1, 2, \cdots, n-1.$$

In this case, we say that a and b cannot be linked. Let

$$\mathcal{A} = \{e \in M^*(D) \mid a \text{ and } e \text{ can be linked}\},$$

$$\mathcal{B} = \{e \in M^*(D) \mid a \text{ and } e \text{ cannot be linked}\},$$

Then $\forall c \in \mathcal{A}$ and $\forall d \in \mathcal{B}$, we have that

$$D' \vee P(c) \vee P(d) = \underline{1}.$$

Put

$$A = \bigwedge\{P(c) \mid c \in \mathcal{A}\}, \quad B = \bigwedge\{P(d) \mid d \in \mathcal{B}\}.$$

Obviously we have that

$$D \not\leq A, D \not\leq B, D' \vee A \vee B = \underline{1}, D \wedge A \wedge B = \underline{0}.$$

This shows that D is not P2-connected. The necessity is proved. □

3. The relation among a few kinds of connectedness

Since a closed set must be a preclosed set, from Definition 1.4 we can obtain the following result.

Theorem 3.1. *A P2-connected L-fuzzzy set must be connected.*

In general a connected L-fuzzy set needn't be P2-connected. This can be seen from the following example.

Example 3.2. Let $X_1 \cap X_2 = \emptyset$, $X = X_1 \cup X_2$, $L = [0,1]$. Defined fuzzy set $(C_a, C_b) \in [0,1]^X$ as follows:

$$(C_a, C_b)(x) = \begin{cases} a, & x \in X_1; \\ b, & x \in X_2. \end{cases}$$

Take

$$\gamma = \{(C_{0.6}, C_0), (C_0, C_{0.6}), (C_{0.5}, C_1), (C_1, C_{0.5}), (C_{0.3}, C_1), (C_1, C_{0.3})\}.$$

Let δ be the [0,1]-topology generated by γ on X. It is easy to see $\underline{1}$ is connected. But it is not P2-connected because there exist two preclosed sets (C_1, C_0) and (C_0, C_1) such that $\underline{1} \not\leq (C_1, C_0)$, $\underline{1} \not\leq (C_0, C_1), \underline{1}' \vee (C_1, C_0) \vee (C_0, C_1) = \underline{1}$, $\underline{1} \wedge (C_1, C_0) \wedge (C_0, C_1) = \underline{0}$.

In order to discuss the relation between P-connectedness and P2-connectedness, first we present two characterizations of P-connectedness.

Theorem 3.3. *Let (X, δ) be an L-space, $D \in L^X$. Then the following conditions are equivalent.*

(1) *D is P-connected.*

(2) *There don't exist preclosed sets A, B such that*

$$D \wedge A \neq \underline{0},\ D \wedge B \neq \underline{0},\ D \leq A \vee B,\ D \wedge A \wedge B = \underline{0}.$$

(3) *There don't exist preclosed sets A, B such that*

$$D \not\leq A,\ D \not\leq B,\ D \leq A \vee B,\ D \wedge A \wedge B = \underline{0}.$$

Proof. (1) $\Rightarrow$ (2). Suppose that there exist preclosed sets A, B such that

$$D \wedge A \neq \underline{0},\ D \wedge B \neq \underline{0},\ D \leq A \vee B,\ D \wedge A \wedge B = \underline{0}.$$

Let $A_1 = D \wedge A$ and $B_1 = D \wedge B$. Then obviously we have that

$$A_1 \neq \underline{0},\ B_1 \neq \underline{0}, D = A_1 \vee B_1,\ (A_1 \wedge pcl(B_1)) \vee (pcl(A_1) \wedge B_1) = \underline{0}.$$

This shows that D is not P-connected.

(2) $\Rightarrow$ (1). Suppose that D is not P-connected. Then there exist two non-null sets A, B such that $D = A \vee B$ and A, B are P-separated. Let $A_1 = pcl(A)$ and $B_1 = pcl(B)$. Then obviously we have that

$$D \wedge A_1 = A \neq \underline{0},\ D \wedge B_1 = B \neq \underline{0}, D \leq A_1 \vee B_1,$$

and

$$D \wedge A_1 \wedge B_1 = (A_1 \wedge pcl(B_1)) \vee (pcl(A_1) \wedge B_1) = \underline{0}.$$

This shows that (2) $\Rightarrow$ (1) is true.

(2) $\Leftrightarrow$ (3) is obvious. □

Theorem 3.4. *Let $1 \in M(L)$. If D is P-connected, then it also is P2-connected.*

Proof. Suppose that D is not P2-connected. Then there exist preclosed sets A, B such that

$$D \wedge A \neq \underline{0},\ D \wedge B \neq \underline{0},\ D' \vee A \vee B = \underline{1},\ D \wedge A \wedge B = \underline{0}.$$

By $D' \vee A \vee B = \underline{1}$ and Lemma 1.3 we can obtain that $D \leq A \vee B$. This shows that D is not P-connected. The proof is obtained. □

Theorem 3.5. *Let D be a crisp subset in (X, δ). Then D is P-connected if and only if it is P2-connected.*

Proof. This can be obtained from the following fact. For a crisp subset D,

$$D' \vee A \vee B = \underline{1} \Leftrightarrow D \leq A \vee B.$$

□

In general, P2-connectedness doesn't imply P-connectedness. This can be seen from the following example.

Example 3.6. In Example 2.2 we have seen that $(C_{0.5}, C_{0.5})$ is P2-connected. Now we shall prove that it is not P-connected. In fact, it is easy to see that both $(C_{0.6}, C_0)$ and $(C_0, C_{0.6})$ are preclosed sets and

$$(C_{0.5}, C_{0.5}) \not\leq (C_{0.6}, C_0),\ (C_{0.5}, C_{0.5}) \not\leq (C_0, C_{0.6}),$$

$$(C_{0.5}, C_{0.5}) \leq (C_{0.6}, C_0) \vee (C_0, C_{0.6}),\ (C_{0.5}, C_{0.5}) \wedge (C_{0.6}, C_0) \wedge (C_0, C_{0.6}) = \underline{0}.$$

Therefore $(C_{0.5}, C_{0.5})$ is not P-connected.

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